**Semiclassical Conductance Fluctuations**

So our classical expectations for the conductance fluctuations are unfounded, because they assume the observable (conductance) is not correlated over the length of the sample, and that we can indeed split the sample up into independent pieces. And this is where the argument goes wrong for mesoscopic samples, since the electrons are correlated over the entire length of the sample (by assumption), since the phase breaking length is L itself. In this case, as we increase the length of the sample, we would not, it seems, be promoting a self-averaging of the conductance. And in fact we will find that the variance is unaffected by scaling the sample. We do find a constant variance, <g>2 ~ e2/h. Interestingly, it is also independent of ni, insofar as it is in the metallic regime. It is only a function of its QM symmetry (whether there is an applied B-field or no, among other things). Note that this variance is typically much smaller than the sample’s actual conductance, G, at least for macroscopically sized samples. And this G, in the conducting regime, goes as L, so the *relative* fluctuations would be expected to decrease as a function of L. This phenomenon is called UCF. We can estimate their fluctuations using Landauer’s formula. First, consider the average conductance:



Comparing to Landauer’s formula,



we can conclude:



Putting everything in terms of reflection instead, we have:



and so,



Putting it in terms of the T goes awry – see Maslov notes. But it works pretty well in terms of R. So proceeding, the variance of g would be:



If we represent the reflection amplitude as a path integral – because it is essentially an S matrix element, then



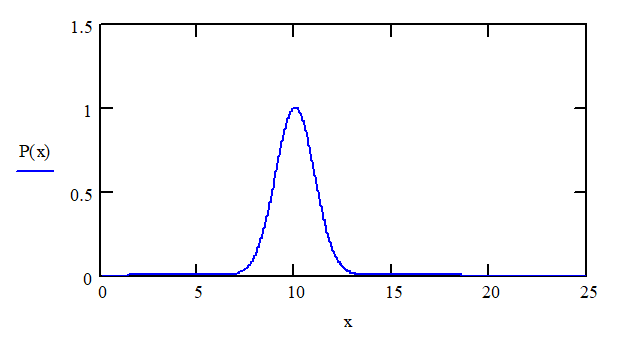
Now we assume that the amplitudes are uncorrelated. This means that for any given distribution of impurities, the value of the amplitude for path α has little or no bearing on the value of the amplitude for path β. Stated another way, the probability that the electron would transmit/reflect via path α has nothing or little to do with the probability that the electron would transmit/reflect via path β. I imagine that this is a bad approximation for the transmittion amplitude because transmition involves many scatterings. A given path therefore samples a large portion of the sample, i.e., gives a lot of information about the total distribution of impurities in the sample. Thus if the amplitude for path α gives us much info on the impurity distribution and therefore has much to do with the amplitude for path β. In contrast, when considering reflection amplitudes, very little information is obtained about the impurity distribution in this way since reflections would occur after only a few collisions. And so the amplitude for one path wouldn’t yield much information towards the amplitude for another path. Anyway, so we have,



So then <g>2 would be:



The prefactor would depend only upon the dimensionality of the sample and its symmetries – spin orbit, TRS, etc. This is an example of universality, where the physical quantity is independent of many of the microscopic details like disorder strength (as measured through τ, say), Fermi wavelength, effective mass, charge carrier density, etc. This of course only holds in the metallic regime, where we can assume that the channels are evenly ‘populated’. So this distribution will look like,



which is quite nicely distributed about the mean, which will be ~ Nℓ/L >> 1 for a metallic system. Now note that if phase breaking mechanisms were present, then the sample would break up into independent sizes on the order of Lφ, and the variance would then self-average according to Vφ/V.